

Effective nucleon-nucleon interaction and low-lying nuclear magnetic states

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Abstract. We present a calculation of low energy magnetic states of doubly-closed-shell nuclei. Our results have been obtained within the random phase approximation using different nucleon-nucleon interactions, having zero- or finite-range and including a possible contribution in the tensor channel.

1. Introduction

In the last thirty years electron-nucleus scattering experiments have produced a large amount of high precision data, which impose severe constraints on nuclear models and effective theories, such as the Random Phase Approximation (RPA). In particular the description of low energy excited states within the RPA is known to be very sensitive to the details of the effective nucleon-nucleon (NN) interaction used in the calculations.

We present here a selection of results from a systematic study of the low energy spectra of several doubly-closed-shell nuclei we have made within the RPA theory [1]. We have focused, in particular, on the unnatural parity states, which are sensitive to the spin, spin-isospin and tensor channels of the residual NN interaction [2].

In the first step of our project we have employed a purely phenomenological approach, using a single-particle mean-field basis generated by a Woods-Saxon well, and constructing phenomenological residual NN interactions which reproduce some selected nuclear states. In order to study the sensitivity of our results to the details of the residual interaction, we have used four different NN interactions, which have zero- and finite-range and may include contributions in the tensor channels. We have found some states which are very sensitive to the details of the interaction, and in general we have obtained a satisfactory description of the low energy states of the nuclei under consideration. We can thus consider our phenomenological approach as an “optimal” RPA approach in terms of comparison with the experimental data.

In our study we have then used the RPA amplitudes X and Y to calculate electron scattering response functions. As a further independent extension of this approach, we have also considered the computation of neutrino cross sections, for which an example will be shown in the following.

We have then proceeded to the second step of our study, performing RPA calculations within a self-consistent approach, where the single-particle mean-field basis is obtained by means of

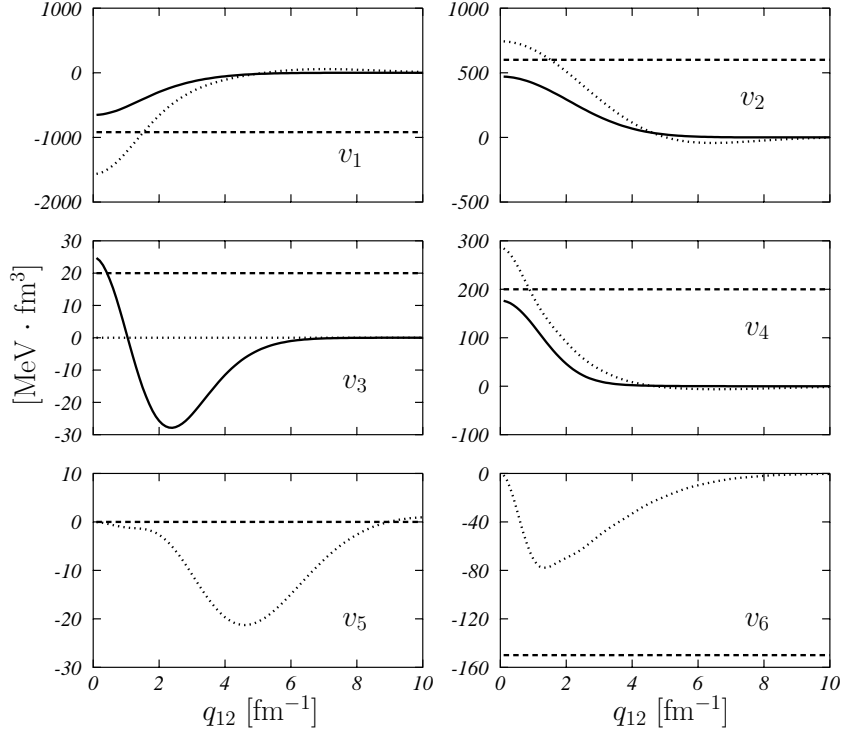


Figure 1. Effective NN interactions as functions of the relative momentum. The dashed lines represent the LMtt interaction, the dotted the FRtt and the solid the Gogny D1. The central channels v_i ($i=1-4$) of the LMtt and FRtt interactions are identical to those of LM and FR, respectively.

a Hartree-Fock calculation which uses the same effective NN interaction used in the RPA calculation. In particular we have used the Gogny D1 finite-range interaction [3, 4, 5], finding, in this case, remarkable disagreement with the experimental spectra.

2. Formalism

The inputs required by RPA calculations are a set of single-particle energies and wave functions and a residual NN interaction.

In the purely phenomenological calculations, for the former we have used the single-particle basis generated by a Woods-Saxon well, whose parameters are taken from the literature [6]. For the latter, we have considered a generic residual interaction written as

$$\begin{aligned}
V_{\text{eff}}(1, 2) = & v_1(r_{12}) + v_1^\rho(r_{12}) \rho^\alpha(r_1, r_2) \\
& + [v_2(r_{12}) + v_2^\rho(r_{12}) \rho^\alpha(r_1, r_2)] \tau_1 \cdot \tau_2 \\
& + v_3(r_{12}) \sigma_1 \cdot \sigma_2 + v_4(r_{12}) \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\
& + v_5(r_{12}) S_{12}(\hat{r}_{12}) + v_6(r_{12}) S_{12}(\hat{r}_{12}) \tau_1 \cdot \tau_2,
\end{aligned} \tag{1}$$

where σ and τ are the usual spin and isospin operators and S_{12} is the tensor operator. As suggested by past phenomenological RPA studies [4, 7] we have included density dependent terms in the the central and isospin channels, with

$$\rho(r_1, r_2) = [\rho(r_1)\rho(r_2)]^{1/2}, \tag{2}$$

and using $\alpha = 1$ in Eq. (1).

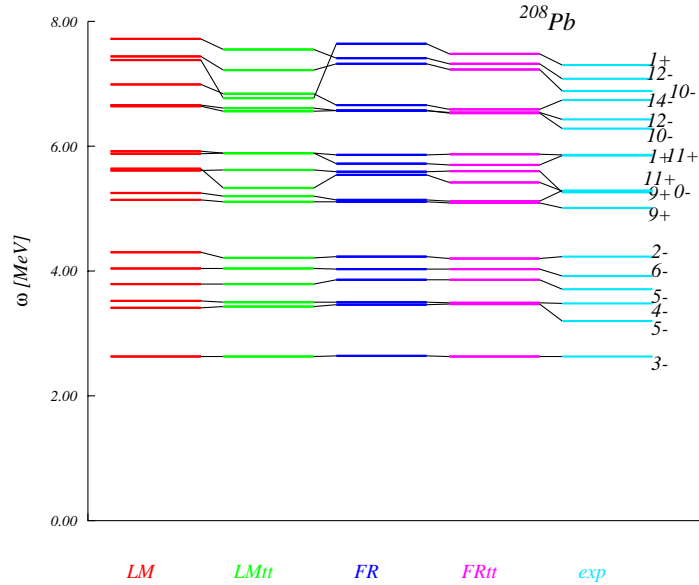


Figure 2. (Color online) Low energy spectrum of ^{208}Pb calculated with the four interactions used in this work and compared with the experimental energies [11].

We have considered four different forms of the NN interaction, parametrizing them according to the following criteria: *(i)* we have chosen a unique set of parameters for all the nuclei under investigation, with the exception of the density dependent terms which are different for each nucleus; *(ii)* the density dependent terms have been set to reproduce the first 2^+ state in ^{12}C and the first 3^- states in the other nuclei; *(iii)* the remaining contributions in the central and isospin channels have been chosen to get a reasonable description of the centroid energy of the isovector giant dipole resonance; *(iv)* the spin, spin-isospin and tensor channels have been adjusted to describe the low energy (below 8 MeV) magnetic spectrum of ^{208}Pb , with particular attention to the 12^- and 1^+ states and, in addition, taking care that the energy of the first 4^- state of ^{16}O is reproduced reasonably.

Following a Landau-Migdal approach, we have first considered zero-range interactions without and with a tensor-isospin channel contribution (LM and LMtt in the following).

We have then constructed finite-range interactions (FR and FRtt), by keeping the long-range behavior of the Argonne v_{18} potential [8] and substituting its short-range part with a sum of Gaussians. We have also used Gaussians to parametrize the density dependent terms of the interaction and, for the FRtt case, we have obtained the tensor channel terms by multiplying the corresponding terms of the v_{18} interaction by a correlation function obtained in variational calculations [6].

The behavior of the various interactions we have considered is shown in figure 1 as a function of the relative momentum of the interacting pair.

In the self-consistent approach we have obtained the single particle basis by solving Hartree-Fock equations [9, 10], using the same effective NN interaction used for the RPA calculations. We have used the Gogny D1 interaction [3, 4, 5], shown in figure 1 by the solid lines, which has finite-range components in the central, isospin, spin and spin-isospin channels and a zero-range density dependent contribution. We have not included the spin-orbit term in the RPA

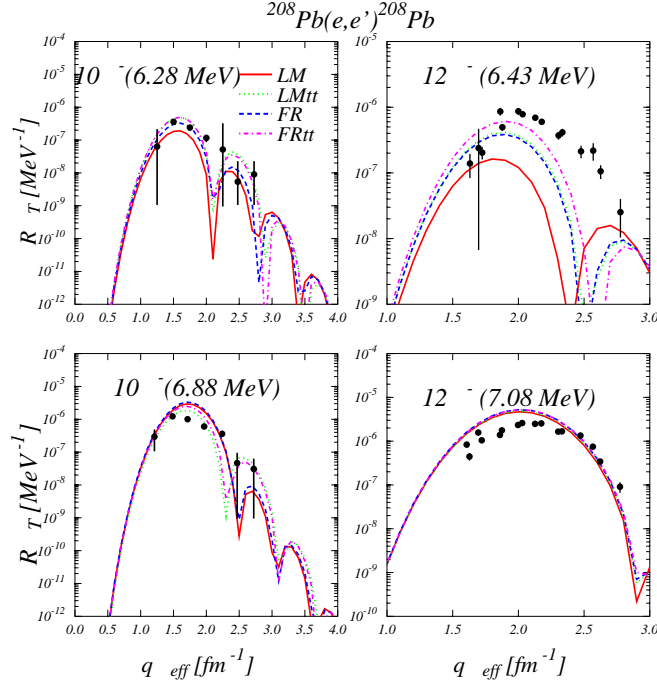


Figure 3. (Color online) Right panels: electron scattering transverse responses for the first 12^- states of ^{208}Pb , versus the effective momentum transfer. Different residual interactions are used as indicated. Left panels: the same for the first 10^- states. Experimental data from [12].

calculations.

For the self-consistent calculations we have included the contributions of both direct and exchange matrix elements of the interaction, whereas for the phenomenological approach we have considered direct terms only, assuming that the effect of the exchange terms is effectively included in the choice of the parameters characterizing the various NN interactions.

Finally we observe that our calculations have been obtained by discretizing the continuum. We have checked that our results are stable with respect to the parameters characterizing the continuum discretization and also with respect to the size of the single-particle configuration space used for each nucleus. More details on the role of the continuum discretization in self-consistent RPA calculations can be found in [2].

3. Results

We have performed systematic calculations of the low energy spectra of ^{12}C , ^{16}O , ^{40}Ca , ^{48}Ca , ^{90}Zr and ^{208}Pb . For each state we have also computed electromagnetic response functions, and we have compared them with the available experimental data.

As an example of low energy spectrum, in figure 2 we show the case of ^{208}Pb . Here we present only those experimental states we have been able to identify with those obtained in our calculations. We observe that the general agreement is quite good and that, except for some cases, the various energies have little sensitivity to the details of the interactions.

The transverse response functions of the 12^- states of ^{208}Pb are shown in the right panels of

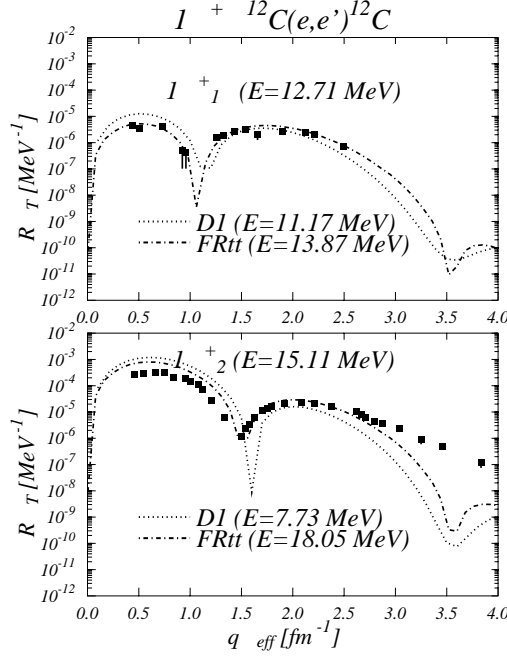


Figure 4. Electron scattering transverse responses of the first 1^+ states of ^{12}C , versus the effective momentum transfer. The self-consistent results obtained with the Gogny D1 interaction are compared with those corresponding to the FRtt phenomenological case. Experimental data from [13, 14].

figure 3. We can see that for the higher state at 7.08 MeV (lower right panel) the experimental data are rather well reproduced with all NN interactions, which do not produce significant differences in the curves. On the other hand, for the lower energy state (upper right panel) we observe a very strong dependence on the residual interaction, both when finite-range and when tensor channel contributions are included. We remark that a better description of this state alone could be obtained with a different choice of the parameters of the residual interaction, but this would worsen the global description of the various magnetic spectra we have considered.

Table 1. Energies of the low-lying magnetic states of ^{12}C (in MeV). Experimental energies from [11].

^{12}C			
excitation	D1	FRtt	exp
1^+	11.17	13.87	12.71
1^+	7.73	18.05	15.11
4^-	19.16	17.75	18.27
4^-	15.63	19.49	19.15

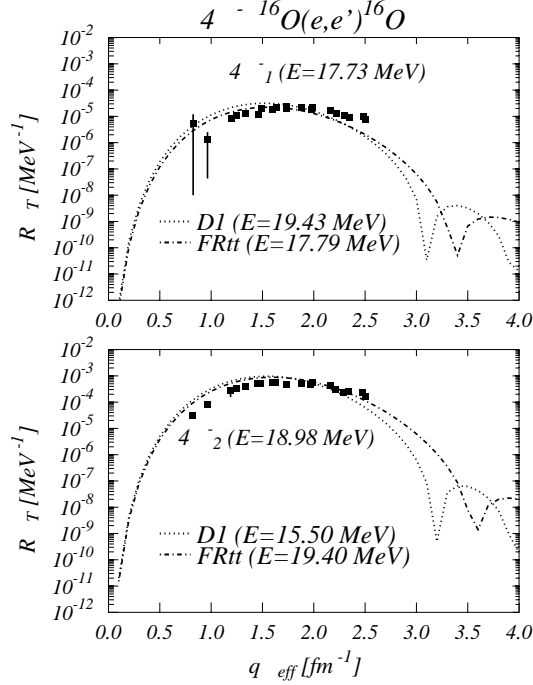


Figure 5. Same as figure 4 for the first 4^- states of ^{16}O . Experimental data from [15].

In the left panels of the same figure the transverse responses of the 10^- states are also shown. It is interesting to notice that, in this case, only the curves which include tensor contributions are able to reproduce the second peak shown by the data, for both states.

An example of the results we have obtained within the self-consistent approach is given in table 1, where the low energy magnetic spectrum of ^{12}C is presented, and in figure 4, where we show the corresponding transverse responses of the 1^+ isospin-doublet of states. For the sake of comparison between the self-consistent results and the purely phenomenological ones, we have taken as reference the FRtt case, considered to be the most complete interaction we have obtained.

We see that the magnetic states of ^{12}C are reproduced rather badly by the self-consistent calculations: the response functions agree in shape and magnitude with the data, but the order of the states forming isospin doublets is inverted.

This happens also for ^{16}O , as illustrated, for the case of the 4^- states, in figure 5, where the values of the excitation energies obtained with the D1 and FRtt interactions are also reported.

We have systematically obtained this kind of inversion for all magnetic states of all the nuclei we have studied. This result indicates the inadequacy of the D1 interaction in isospin-dependent channels, and it may be considered the most important outcome of the self-consistent part of our study.

4. Extension of the phenomenological approach to neutrino scattering

After obtaining an “optimal” phenomenological RPA approach, by tuning NN interactions to reproduce data from electron scattering at best, we have also considered the possibility of

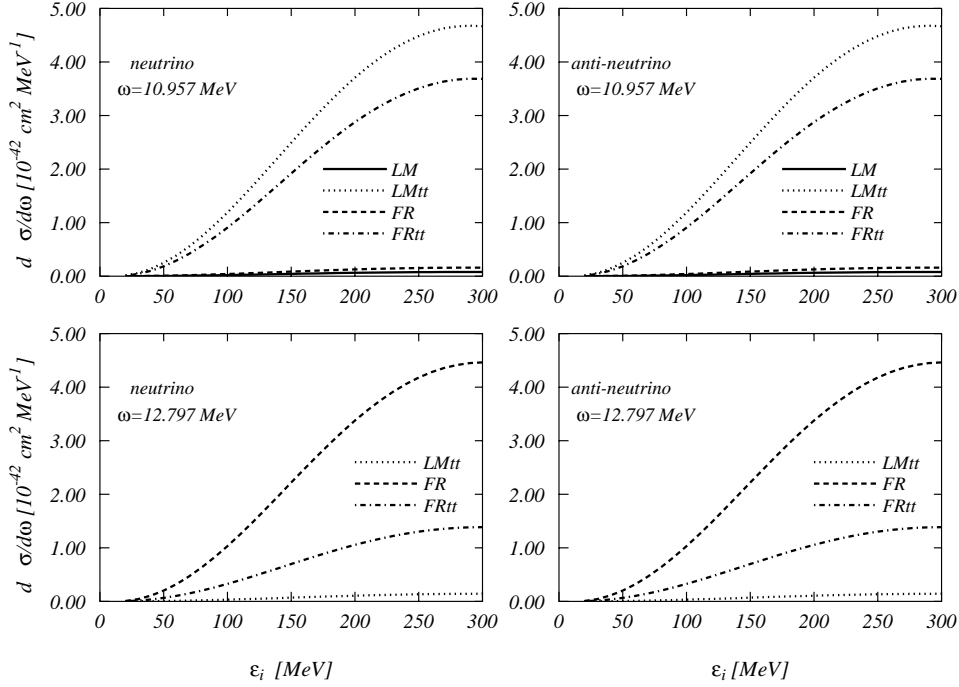


Figure 6. Cross sections for the neutral current neutrino and antineutrino excitation of 0^- states of ^{16}O at excitation energy ω , versus the incident (anti)neutrino energy.

applying it to the study of low energy neutrino scattering cross-sections, with the purpose of studying their sensitivity to the tensor components of the interaction.

In figure 6 we show the cross sections for the neutral-current neutrino and antineutrino excitation of the low energy 0^- states of ^{16}O , as a function of the incident neutrino energy. The values of the excitation energies ω are indicated inside the figure.

We observe that, in this test-case, extremely large differences in the cross sections are obtained when tensor channel contributions are included in the effective NN interaction. This indicates that the role of the residual interaction in neutrino scattering cross section is a very interesting topic, for example in connection with the problem of nuclear uncertainties in the detection of supernova neutrinos [16].

5. Summary and conclusions

We have made a systematic study of the low-lying (magnetic) spectra of doubly-closed-shell nuclei. Our study indicates that a simultaneous description of all states imposes strong constraints on the residual NN interaction. Within a purely phenomenological approach it is possible to get a good description of the spectra and of the response functions of most of the states, thus obtaining an “optimal” RPA approach. Some states which are not well described exhibit a strong sensitivity to some details of the residual interaction, and a deeper investigation could be used to obtain further constraints on it.

On the contrary, the self-consistent calculations we have performed with the Gogny D1 interaction produce results in disagreement with the experimental spectra, indicating the

presence of problems in particular in the isospin dependent parts of the interaction. A detailed study can be found in [2].

Finally we have considered an example of a potentially important application of our phenomenological approach to the calculation of neutrino scattering cross sections, to investigate the sensitivity of the latter to the tensor components of the residual interaction.

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