Application of BCS theory to nuclear systems

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The pairing phenomenon is a particular feature of fermionic many-bodies systems. The fermions in these systems combine to form couples of particles, characterized by zero angular momentum.

Bosonic couples are at the basis of the description of the superconducting metals, which is part of BCS theory, named after Bardeen, Cooper and Schrieffer.

Nuclei can be described by using the BCS theory, as they also consist of pairs of nucleons, named Cooper's couples. The nuclear structure exhibits many similarities with the electron structure of metals. In both cases, we are dealing with systems of fermions which may be described in first approximation in terms of independent particle motion.

Moreover, differently from superconducting metals, atomic nuclei cannot be considered infinite systems. It is necessary to use the BCS equations, suitable for finite systems, characterized by rotational invariance.

The BCS vacuum does not possess good particle number. In standard BCS theory the ground-state expectation value of the particle number operator is constrained to be the desired number of particles. Imposing the constraint on the average particle number leads to a *constrained variational problem.* We constructed a code to solve the BCS equations.

We tested the stability of the results with respect to the choice of some physics parameters, such as the size of the single particle configuration space. We made these tests by studying the ^{18}O nucleus.

In our analysis, we used three different types of pairing interactions:

- schematic interaction G [1];
- zero range interaction δ;
- finite range interaction F.

The tests of convergence show that the finite range interaction converges more quickly than the others. The zero range interaction δ has a slower convergence, while G doesn't converge at all.

We have studied the properties of the Ca even-even isotopic chain, focusing on the evolution of pairing properties from the ^{40}Ca closed shell nucleus to the other closed shell nucleus ^{48}Ca.

In Fig. (1) we show the results of the pairing densities of the Ca isotopes, obtained with finite range interaction. The pairing density is defined as [2]:

$$\kappa(\mathbf{r}) = \langle \psi(\mathbf{r}, s = 1/2)\, \psi(\mathbf{r}, s = -1/2) \rangle \ , \tag{1}$$

where the operator $\psi(\mathbf{r}, s = 1/2)$ annichilates at the point $\mathbf{r}$ a nucleon with the spin projection s. The pairing density is indicative of the probability to find a couple of nucleons with coupled spin.

In spherical nuclei the pairing density can be expressed as:

$$\kappa(\mathbf{r}) = \sum_{nlj} |u_{\rm nlj}|\, |v_{\rm nlj}|\, R^2_{\rm nlj}(r)\, \frac{2j+1}{4\,\pi}, \tag{2}$$

where $u_{\rm nlj}$ and $v_{\rm nlj}$ are variational parameters which are normalized as

$$|u_{\rm k}|^2 + |v_{\rm k}|^2 = 1 \ . \tag{3}$$

The physical interpretation of the variational parameters is that $u^2_{\rm nlj}$ represent the probability that the level characterized by the quantum numbers nlj is occupied, and $v^2_{\rm nlj}$ is the probabilitity that this level is empty [3].

The global effect of the pairing is to increase energy gap, that is the energy difference between the last occupied level below the Fermi level, and the first empty level above the Fermi surface.

The results of Fig. 1 confirm that the term of the pairing doesn't contribute to the closed shells. In fact, for the double magic nuclei ^{40}Ca and ^{48}Ca the pairing density is almost two orders of magnitude smaller than those of the other nuclei, as we show in the panel (b) of the figure.

The comparison in linear scale, panel (a) of the figure, shows that the greater values of the pairing densitie are those of ^{44}Ca nucleus which is farest one from the two doubly magic nuclei ^{40}Ca and ^{48}Ca of all the nuclei we have investigated. This result indicates that the effect of the pairing is more relevant in nuclei distant from the shell closure.

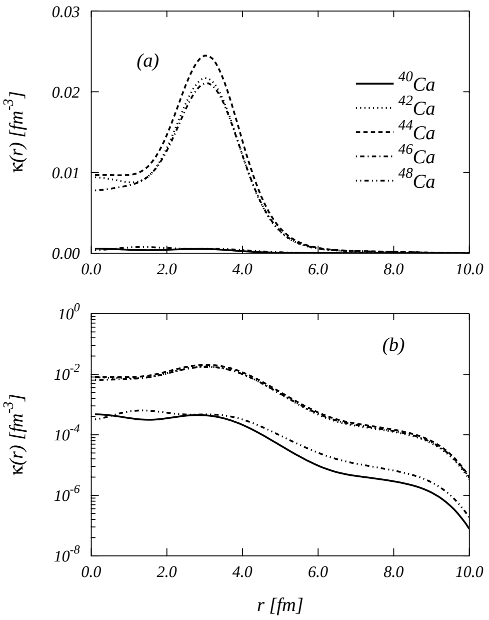

Figure 1. Pairing density for Ca isotopes, obtained with finite range interaction. The same results have shown both in linear scale in the panel (a), and in logarithmic scale in the panel (b).

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